Identità trigonometriche:

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\cos\alpha\cos\beta=1/2(\cos(\alpha+\beta)+\cos(\alpha-\beta))$

$\sin\alpha\sin\beta=1/2(\cos(\alpha-\beta)-\cos(\alpha+\beta))$

$\sin\alpha\cos\beta=1/2(\sin(\alpha+\beta)+\sin(\alpha-\beta))$

$\sin\alpha+\sin\beta=2\sin[1/2(\alpha+\beta)]\cos[1/2(\alpha-\beta)]$

$\cos\alpha+\cos\beta=2\cos[1/2(\alpha+\beta)]\cos[1/2(\alpha-\beta)]$

$\sin(-\alpha)=-\sin\alpha$ $\qquad$ $\cos(-\alpha)=\cos\alpha$

$\sin(2\alpha)=2\sin\alpha\cos\alpha$ $\qquad$ $\cos(2\alpha)=2\cos^2\alpha-1$

$\sin\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1}{2}(1-\cos\alpha)}$ $\qquad$ $\cos\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1}{2}(1+\cos\alpha)}$

$\cos^2\alpha=1/2(\cos(2\alpha)+1)$ $\qquad$ $\sin^2\alpha=1-\cos^2\alpha$

Formule di Eulero:

$\sin\alpha=\frac{e^{j\alpha}-e^{-j\alpha}}{2j}$ $\qquad$ $\cos\alpha=\frac{e^{j\alpha}+e^{-j\alpha}}{2}$ $\qquad$ $e^{j\alpha}=\cos\alpha+j\sin\alpha$

Integrali notevoli:

$\int e^{\alpha x}dx=\frac{e^{\alpha x}}{\alpha}$

$\int\cos\alpha x\,dx=\frac{\sin\alpha x}{\alpha}$ $\qquad$ $\int\sin\alpha x=-\frac{\cos\alpha x}{\alpha}$

$\int e^{\alpha x}\sin\beta x=\frac{e^{\alpha x}(\alpha\sin\beta x-\beta\cos\beta x)}{\alpha^2+\beta^2}$

$\int e^{\alpha x}\cos\beta x=\frac{e^{\alpha x}(\beta\sin\beta x-\alpha\cos\beta x)}{\alpha^2+\beta^2}$

$\int f(x)g(x)dx=F(x)g(x)-\int g'(x)F(x)dx$

Energia e Potenza:

S.E. $\xi<\infty\rightarrow P_s=0$ $\qquad$ $\xi_s=\int_{-\infty}^{+\infty}|s(t)|^2dt$ $\qquad$ $\xi_s^{RMS}=\sqrt{\xi_s}$

S.P. $\xi_s=0\rightarrow P_s=k$ $\qquad$ $P_s=\lim_{T\to\infty}\frac{1}{2\pi}\int_{-T}^{+T}|s(t)|^2dt$ $\qquad$ $P_s^{RMS}=\sqrt{P_s}$

Per segnali periodici del tipo $s(t)=\sum_{k=-\infty}^{\infty}i(t-kT_0)$ si ha:

$P_s=\frac{1}{T_0}\int_{t_0}^{t_0+T_0}|s(t)|^2dt$ ovvero $P_s=\frac{\xi_i}{T_0}$

Energie e Potenze notevoli:

$s(t)=\alpha e^{-\beta t}u(t)\quad\Rightarrow\quad\mu_s=0;\quad\xi_s=\frac{\alpha^2}{2\beta}$

$s(t)=A\cos(2\pi f_0t+\theta_0)\quad\Rightarrow\quad P_s=\frac{A^2}{2}$

$s(t)=A\Pi\left(\frac{t-t_0}{T}\right)\quad\Rightarrow\quad\xi_s=A^2T\qquad da:\ t_0-\frac{T}{2},t_0+\frac{T}{2}$

$s(t)=A\Lambda\left(\frac{t-t_0}{T}\right)\quad\Rightarrow\quad\xi_s=A^2\frac{2}{3}T\qquad da:\ t_0-T,t_0+T$

Decibell:

$X_{dB}=10\log_{10}X$ $\quad$ <=> $\quad$ $X=10^{\frac{X_{dB}}{10}}$

$X_{dBr}=10\log_{10}\frac{X}{X_0}$ $\quad$ <=> $\quad$ $X_{dBr}=20\log_{10}\frac{X^{RMS}}{X_0^{RMS}}$

Se $y(t)=A\,s(t)$ $\qquad$ *guad.* $A_{dB}=10\log_{10}A^2$

Serie di Fourier:

funzione di costo: $\xi=\int_a^b|\hat{s}(t)-s(t)|^2dt$

Coseno: $\hat{s}(t)=a_0+\sum_{n=1}^{\infty}a_n\cos(2\pi\frac{n}{T}t)$

$a_0=\frac{1}{T}\int_{-T/2}^{T/2}s(t)dt$ $\qquad$ $a_n=\frac{2}{T}\int_{-T/2}^{T/2}s(t)\cos(2\pi\frac{n}{T}t)dt$

Seno: $\hat{s}(t)=\sum_{n=1}^{\infty}b_n\cos(2\pi\frac{n}{T}t)$ $\qquad$ $b_n=\frac{2}{T}\int_{-T/2}^{T/2}s(t)\sin(2\pi\frac{n}{T}t)dt$

Trigonometrica: $\hat{s}(t)=a_0+\sum_{n=1}^{\infty}a_n\cos(2\pi\frac{n}{T}t)+b_n\sin(2\pi\frac{n}{T}t)dt$

Compatta: $\hat{s}(t)=C_0+\sum_{n=1}^{\infty}C_n\cos(2\pi\frac{n}{T}t+\theta_n)$

con $C_0=a_0$ e $C_n=\sqrt{a_n^2+b_n^2}$ e $\theta_n=\tan^{-1}\left(\frac{-b_n}{a_n}\right)$

Esponenziale: $\hat{s}(t)=\sum_{n=-\infty}^{\infty}C_ne^{j2\pi\frac{n}{T}t}$ $\qquad$ $C_n=\frac{1}{T}\int_{-T/2}^{T/2}s(t)e^{-j2\pi\frac{n}{T}t}dt$

se *s(t)* è reale: $C_n=C_{-n}^{\circ}$ $\qquad$ se *s(t)* è pari: $C_n=costante\ reale$

Trasformata di Fourier:

$S(f)=\int_{-\infty}^{+\infty}s(t)e^{-j2\pi ft}dt$ $\qquad$ $s(t)=\int_{-\infty}^{+\infty}S(f)e^{j2\pi ft}df$

Trasl: $F[s(t-t_0)]=e^{-j2\pi ft_0}S(f)$ $\qquad$ Dualità: $F[S(t)]=s(-f)$

Scala: $F[s(at)]=\frac{1}{|a|}S\left(\frac{f}{a}\right)$ $\qquad$ Der: $F\left[\frac{d^n}{dt^n}s(t)\right]=(j2\pi f)^nS(f)$

Integrale: $F\left[\int_{-\infty}^{t}s(\xi)d\xi\right]=\frac{1}{j2\pi f}S(f)$

Teorema di Parseval: $\int_{-\infty}^{+\infty}x(t)y^{*}(t)dt=\int_{-\infty}^{+\infty}X(f)Y^{*}(f)df$

se s(t) reale: S(f) Hermit. $|S(f)|=|S(-f)|$ e $\widehat{S(f)}=-\widehat{S(-f)}$

Trasformate notevoli:

$F[\delta(t)]=1$

$F[A]=A\delta(f)$

$F\left[A\Pi\left(\frac{t-t_0}{T}\right)\right]=AT\,sinc(Tf)e^{-j2\pi ft_0}$

$F\left[A\Lambda\left(\frac{t}{T}\right)\right]=AT\,sinc^2(Tf)e^{-j2\pi ft_0}$

$F\left[\sum_{k=-\infty}^{+\infty}\delta(t-kT)\right]=\frac{1}{T}\sum_{n=-\infty}^{+\infty}\delta\left(f-\frac{n}{T}\right)$

$F\left[\sum_{k=-\infty}^{+\infty}i(t-kT)\right]=I(f)\left(\frac{1}{T}\sum_{n=-\infty}^{+\infty}\delta\left(f-\frac{n}{T}\right)\right)$

$F[u(t)]=\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$ $\qquad$ $F[sgn(t)]=\frac{1}{j\pi f}$

$F[ke^{-at^2}]=k\sqrt{\frac{\pi}{a}}e^{\frac{-\pi^2}{a}f^2}$ $\qquad$ $F[t^ne^{-at}u(t)]=\frac{n!}{(a+j2\pi f)^{n+1}}$

$F[\cos(2\pi f_0t+\theta)]=\frac{1}{2}(e^{j\theta}\delta(f-f_0)+e^{-j\theta}\delta(f+f_0))$

$F[\sin(2\pi f_0t+\theta)]=\frac{1}{2j}(e^{j\theta}\delta(f-f_0)-e^{-j\theta}\delta(f+f_0))$

Spettri di energia e potenza:

$\xi_s(f)=|S(f)|^2$ $\qquad$ $P_s(f)\approx\lim_{T\to\infty}\frac{|s_T(f)|^2}{2T}$

$s(t)=Acos(2\pi f_0t+\theta)\ \Rightarrow\ P_s(f)=\frac{A^2}{4}[\delta(f-f_0)+\delta(f+f_0)]$

per segnali periodici $P_s(f)=\frac{1}{T_0^2}|I(f)|^2\sum_{k=-\infty}^{+\infty}\delta\left(f-\frac{k}{T_0}\right)$

Autocorrelazione e Mutua correlazione:

S.E. $F[r_s(t)]=\xi_s(f)$ $\qquad$ $r_s(\tau)=\int_{-\infty}^{+\infty}s(t)s^{*}(t-\tau)dt$

$r_s(t)=s(\tau)*s^{*}(-\tau)$ $\qquad$ $r_s(0)=\xi_s$

S.P. $F[r_s(t)]=P_s(f)$ $\qquad$ $r_s(0)=P_s$

S.E. $r_{xy}(\tau)=\int_{-\infty}^{+\infty}x(t)y^{*}(t-\tau)dt$ $\qquad$ $\xi_{xy}(f)=X(f)Y^{*}(f)$

$F[r_{xy}(t)]=\xi_{xy}(f)$

S.P. $r_{xy}(\tau)=\lim_{t\to+\infty}\frac{1}{2T}\int_{-\infty}^{+\infty}x(t)y^{*}(t-\tau)dt$ $\qquad$ $F[r_{xy}(t)]=P_{xy}(f)$

Se $r_{xy}(0)=0\ \Rightarrow\ x(t)\perp y(t)$

Se $r_{xy}(\tau)=0\ \Rightarrow\ x(t)\,e\,y(t)\,incoerenti$

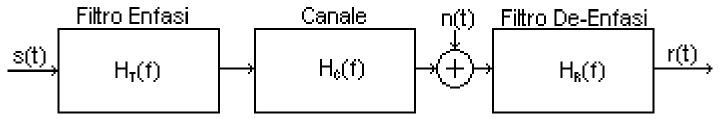
$r_{x+y}(\tau)=r_x(\tau)+r_y(\tau)+r_{xy}(\tau)+r_{yx}(\tau)$

Se $x(t)\perp y(t)\ \Rightarrow\ r_{x+y}(0)=r_x(0)+r_y(0)$

Se $x\,e\,y\,inc.\Rightarrow r_{xy}(\tau)=r_x(\tau)+r_y(\tau)$

Realizzato da *Enrico Saviano* http://enrico.saviano.org

Filtri



se $K=|H_R(f)||H_c(f)||H_T(f)|$ (con $|H_c(f)|=1$ se c. nn dist.)

allora: $SN_R=\frac{P_{s_R}}{P_{n_R}}=\frac{K^2P_s}{\int_{-\infty}^{+\infty}|H_R(f)|^2P_n(f)\,df}$ da cui si ha:

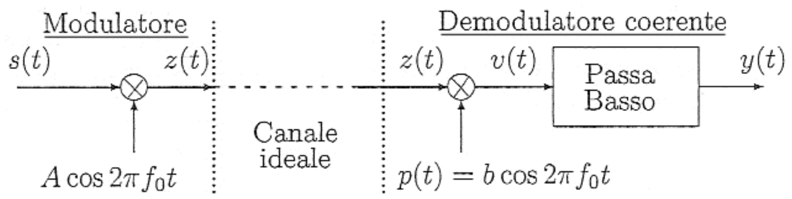
$$|H_{T0}(f)|^2=\frac{\alpha K}{|H_c(f)|}\frac{\sqrt{P_n(f)}}{\sqrt{P_s(f)}} \qquad |H_{R0}(f)|^2=\frac{K}{\alpha|H_c(f)|}\frac{\sqrt{P_s(f)}}{\sqrt{P_n(f)}}$$

Modulazione

$\eta_s=\frac{\text{banda occupata segnale modulante}}{\text{banda occupata segnale modulato}}$ efficienza spettrale

Segnale modulato **DSB**: $z(t)=A\,s(t)\cos 2\pi f_0 t$

Fourier s. modulato: $Z(f)=\frac{A}{2}[\delta(f-f_0)+\delta(f+f_0)]$

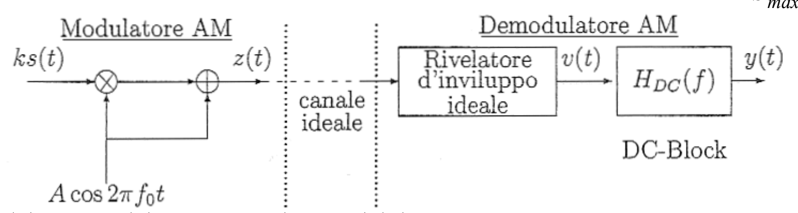


$y(t)=\frac{Ab}{2}s(t)$ $\qquad \bar{P}_z=\frac{A^2}{2}P_s$ $\qquad \eta_{s_{DSB}}=0.5$

Scostamento in fase: $y(t)=\frac{Ab}{2}s(t)\cos\Phi_0$ (segnale attenuato)

Scostamento in freq: $y(t)=\frac{Ab}{2}s(t)\cos 2\pi\Delta t$ $_{LPF(-B,B)}$ (mod residua)

Segnale modulato **AM**: $z(t)=A(1+ks(t))\cos 2\pi f_0 t$ con $k\leq\frac{1}{s_{max}}$



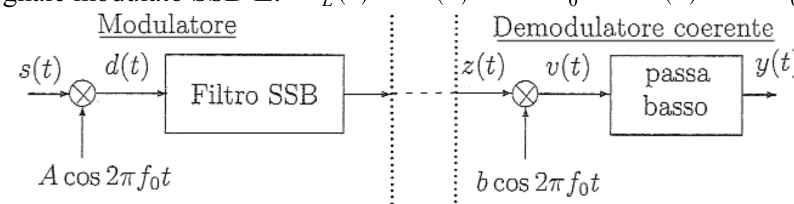
$y(t)=A\,ks(t)$ $_{H_{DC}}=Ak(h_{dc}*s)(t)$ (non comporta dist. appr.)

$\bar{P}_z=\bar{P}_p+\bar{P}_b$ $\qquad \bar{P}_p=\frac{A^2}{2}$ $\qquad \bar{P}_b=\frac{A^2k^2}{2}P_s$ $\qquad \eta_{s_{AM}}=0.5$

$\eta_p=\frac{\bar{P}_b}{\bar{P}_p+\bar{P}_b}=\left(1+\frac{1}{k^2P_s}\right)^{-1}$ efficienza di potenza

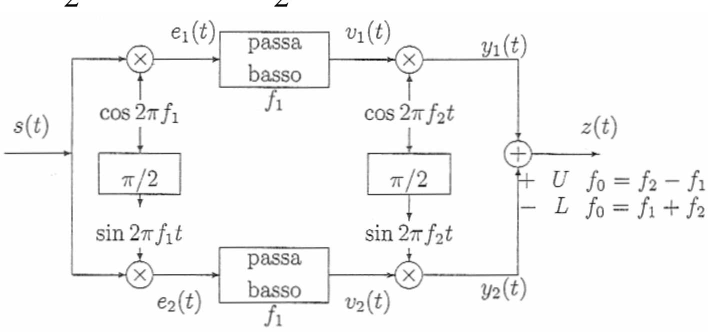
Segnale modulato **SSB-U**: $z_U(t)=As(t)\cos 2\pi f_0 t-A\hat{s}(t)\sin 2\pi f_0 t$

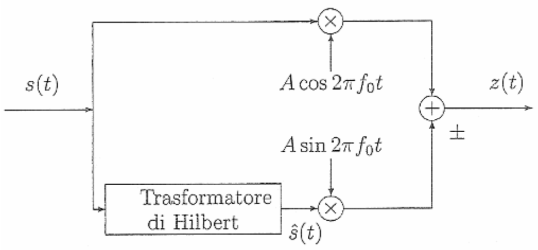
Segnale modulato **SSB-L**: $z_L(t)=As(t)\cos 2\pi f_0 t+A\hat{s}(t)\sin 2\pi f_0 t$



$y(t)=\frac{Ab}{2}s(t)$ $\qquad P_z=A^2P_s$ $\qquad \eta_{s_{SSB}}=1$

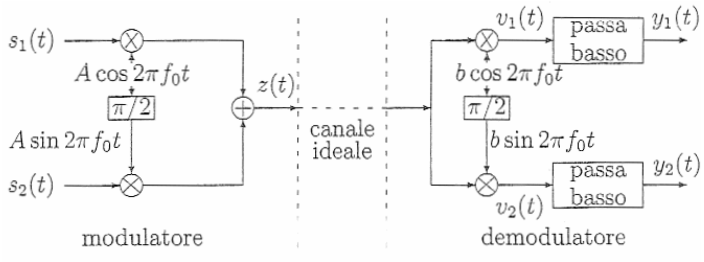
$P_z(f)=\frac{A^2}{2}P_s(f-f_0)+\frac{A^2}{2}P_s(f+f_0)$





VSB: Idem che SSB ma con filtro $H_{VSB}(f)$ $\qquad \hat{P}_z=\frac{A^2}{2}(P_s+P_{\check{s}})$

Segnale modulato **QAM**: $z(t)=As_1(t)\cos 2\pi f_0 t+As_2(t)\sin 2\pi f_0 t$



$y_1(t)=\frac{Ab}{2}s_1(t)$ $\qquad y_2(t)=\frac{Ab}{2}s_2(t)$ $\qquad \eta_{s_{QAM}}=1$

con scostamento in fase $y_1(t)=\frac{Ab}{2}s_1(t)\cos\theta-\frac{Ab}{2}s_2(t)\sin\theta$

$y_2(t)=\frac{Ab}{2}s_1(t)\sin\theta+\frac{Ab}{2}s_2(t)\cos\theta$

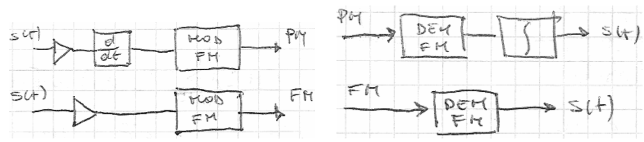
Modulazione Angolare: $z(t)=A\cos(2\pi f_0 t+\Psi(t))$

Phase Mod: $\Psi(t)=s_p s(t)$ $\qquad$ Freq. Mod: $\Psi(t)=s_f\int^t s(\xi)\,d\xi$

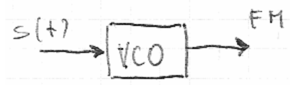
PM: $f_z(t)=f_0+\frac{s_p}{2\pi}\dot{s}(t)$ $\qquad \Delta f_z(t)=\frac{s_p}{2\pi}\dot{s}(t)$

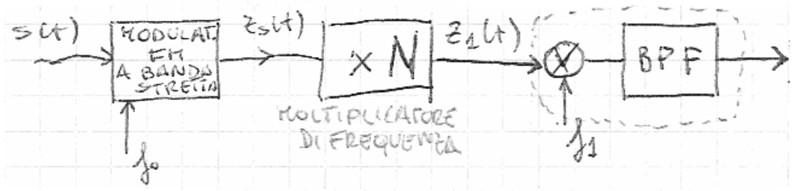
FM.: $f_z(t)=f_0+\frac{s_f}{2\pi}s(t)$ $\qquad \Delta f_z(t)=\frac{s_f}{2\pi}s(t)$

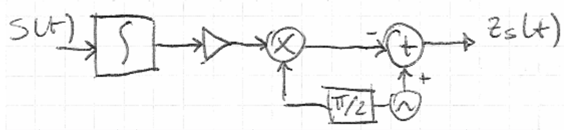
Banda di Carson: $W=2B(1+m)$ con $m=\frac{\Delta f_{z,MAX}}{B}$ indice di mod.

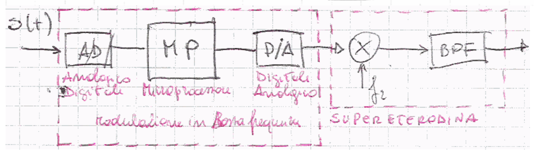


Modulatori FM

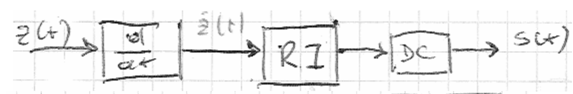
metodo diretto:  (oscillatore a freq. variabile)

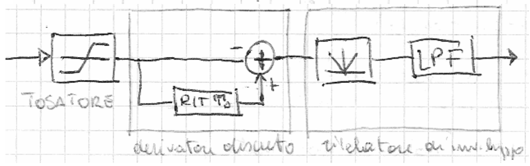
indir: 

mod. di Armstrong: 

metodo digitale: 

Demodulatori FM

Discriminatore di freq. 

Alternativa: 

Comportamento modulazioni in presenza di rumore

DSB:	$\bar{P}_z=\frac{A^2}{2}P_s$	$P_n=2\eta_0 B$	$(S/N)_{in}=\frac{A^2}{4}\frac{P_s}{\eta_0 B}$
AM:	$\bar{P}_z=\frac{A^2}{2}(1+k^2P_s)$	$P_n=2\eta_0 B$	$(S/N)_{in}=\frac{A^2}{4}\frac{1+k^2P_s}{\eta_0 B}$
SSB:	$\bar{P}_z=\frac{A^2}{2}P_s$	$P_n=\eta_0 B$	$(S/N)_{in}=\frac{A^2P_s}{\eta_0 B}$
VSB:	$\bar{P}_z=\frac{A^2}{2}(P_s+P_{\check{s}})$	$P_n=(\frac{\epsilon}{2}+B)\eta_0$	$(S/N)_{in}=\frac{A^2}{2}\frac{P_s+P_{\check{s}}}{(\frac{\epsilon}{2}+B)\eta_0}$
QAM:	$\bar{P}_z=\frac{A^2}{2}(P_{s_1}+P_{s_2})$	$P_n=2\eta_0 B$	$(S/N)_{in}=\frac{A^2}{4}\frac{P_{s_1}+P_{s_2}}{\eta_0 B}$

DSB:	$P_{z_c}=A^2P_s$	$P_{n_c}=2\eta_0 B$	$(S/N)_{out}=\frac{A^2P_s}{2\eta_0 B}$
AM:	$P_{z_{c}(DC)}=A^2k^2P_s$	$P_{n_c(DC)}=2\eta_0 B$	$(S/N)_{out}=\frac{A^2k^2P_s}{2\eta_0 B}$
SSB:	$P_{z_c}=A^2P_s$	$P_{n_c}=\eta_0 B$	$(S/N)_{out}=\frac{A^2P_s}{\eta_0 B}$
VSB:	$P_{z_c}=A^2P_s$	$P_{n_c}=(\frac{\epsilon}{2}+B)\eta_0$	$(S/N)_{out}=\frac{A^2P_s}{(\frac{\epsilon}{2}+B)\eta_0}$
QAM:			$(S/N)_{out\ i}=\frac{A^2P_{s_i}}{2\eta_0 B}$